



FIG. 11. Percent permanent bore enlargement for 100 per cent overstrain vs. diameter ratio.

The elastic recovery at the bore is given by the Lamé equations and Eq. (4).

$$\epsilon_{tae} = \frac{\sigma_y}{E} \frac{1.08 \ln W}{W^2 - 1} [(1 + \mu) W^2 + (1 - \mu)] \quad (27)$$

Subtracting Eq. (27) from Eq. (26) yields:

$$\epsilon_{taperm} = [\mu + (2 - \mu)W^2] \left[\frac{1}{2} - \frac{1.08 \ln W}{W^2 - 1} \right] \quad (28)$$

This equation is plotted in Fig. 11 for $\mu = 0.3$ and is in good agreement with the experimental values.

SUMMARY

The application of the autofrettage principle to materials of 165,000 psi yield strength has been experimentally investigated. Resulting from this investigation are a series of relationships for the stresses and displacements in overstrained thick-wall cylinders in the open-end condition. These relationships, which can be used for pressure vessel design purposes, are based on the von Mises yield criterion but incorporating empirical constants for simplicity. The agreement between these relationships and the experimental data is good.

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